2022

Time : As in Programme

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer **ALL** *questions.*

PART - I

1. Answer all the questions. $1 \times 12 = 12$

(a) $\lim\limits_{(x,y)\to(0,1)} \dfrac{x+y-1}{\sqrt{x+y-1}}$ is ________.

(b) If $u = x(1-x),\ v = xy$ then $\dfrac{\partial(u,v)}{\partial(x,y)}$ is ________.

(c) If u is a homogenous function of degree 3, then

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \_\_\_\_\_\_\_\_.$$

(d) If $f(x,y) = x^2 + y^2 + 3$ then f has extreme value at ________.

(e) If $z = \log(x^{\to 2} + y^2)$, $x = u + v$, $y = u - v$ then z is a compositive function of ________.

(f) $\int_0^1 \int_0^1 x^2 dxdy =$ ________.

(g) If the repeated limit exist but not equal then what can be said about the simulataneous limit?

(h) Expression of $x+y=3$ in powers of $(x-1)$ and $(y-1)$ is ________.

(i) $z=xy\,f\left(\frac{x}{y}\right)$ then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=$ ________.

(j) If $f(x,y)=x^2-2y^2+1$ then f has extreme values at ________.

(k) If $z=x^3-xy+y^3$, $x=r\,Cos\theta$, $y=r\,Sin\theta$ then $\frac{\partial z}{\partial r}=$ ________.

(l) If $f(x,y)=\frac{2xy}{x^2+y^2}$ then $\lim_{(x,y)\to(0,0)} f(x,y)$ is ________.

2. Answer any eight questions. $2\times 8=16$

(a) Let $z=x^2\,\text{Sin}\left(3x+y^3\right)$ then find $\frac{\partial z}{\partial x}$ $(\pi/3,0)$

(b) If $f(x,y)=x^2y+y^3$ then find $\nabla f(x,y)$

(c) Find the value of $\nabla\left(f^n\right)$

(d) Find the critical points of the function $f(x,y)=8x^3-24xy+y^3$

(e) Evaluate $\iint_R x^2y^5\,dA$ where R is the rectangle $1\le x\le 2,\ 0\le y\le 1$ using an iterated integral with x-integration first.

(f) Express work as a line integral

(g) Is the vector field $\overset{u}{F} = ye^{xy}\hat{e} + (xe^{xy} + x)\hat{j}$ conservative.

(h) Write divergence theorem in the plane.

(i) Find a vector that is normal to the level surface $x^2 + 2xy - yz + 3z^2 = 7$ *at* $(1, 1, -1)$.

(j) Let $f(x, y) = \begin{cases} \dfrac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$

Find $f_y(0, 0) = ?$

PART - III

3. Answer any eight question. $3 \times 8 = 24$

(a) Find the points where the function $f(x, y, z) = \dfrac{3}{\sqrt{x^2 + y^2 - 2z}}$ is discontinuous.

(b) $z = 4x - y^2$ where $x = \vartheta v^2$, $y = \vartheta^2 v$ find $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$.

(c) Find the directional derivative of $f(x, y) = \ln(x^2 + y^3)$ *at* $(1, -3)$ in the direction of $V = 2\hat{i} - 3\hat{j}$.

(d) Find all relative extream and saddle points of the function $f(x, y) = 2x^2 + 2xy + y^2 - 2x - 2y + 5$

(e) Find the volume of the solid bounded above by the plane $z = y$ and below in the xy plane by the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant.

(f) Evaluate $\int_0^1 \int_{x^3}^1 (x+y^2)\,dydx$ by reversing the order.

(g) Show that the Area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is π ab.

(h) Use Green's theorem to find the work done by the force field $f(x,y)=(3y-4x)\hat{i}+(4x-y)\hat{j}$ when an object moves once counter clockwise around the ellipse $4x^2+y^2=4$.

(i) Evaluate the line integral $\int_C x^2 z\, ds$ where C is the helix $x=\cos t,\ y=2t,\ z=\sin t$ for $0 \le t \le \pi$.

(j) Find an equation in cylindrical co-ordinates for the elliptic paraboloid $z=x^2+3y^2$.

PART - IV $7 \times 4 = 28$

4. Show that for $f(x,y)=\sqrt{|xy|}$ both f_x and f_y exist at (0,0) but not differentiable at (0,0)

Or

Let $f(x,y)=\begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$

Show that at Origin $f_{xy} \neq f_{yx}$.

5. Use Lagrange's multipliers to maximize $f(x,y)=x^2-2y-y^2$ subject to $x^2+y^2=1$

Or

Compute the Area of the region D bounded by the line $y=x$ and below by the circle $x^2+y^2-2y=0$

6. Find the volume of the solid D bounded by the paraboloid $z=x^2+y^2$ and above the plane $2x+z=3$.

Or

Evaluate the integral

$$I=\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{\sqrt{2-x^2-y^2}} z\,dz\,dy\,dx$$

7. Show that the vector field

$$F=\left(e^x \,Sin\, y-y\right)\hat{e}+\left(e^x Cos\, y-x-z\right)\hat{j}$$

is conservative and then find a Scalar potential function f for F.

Or

Show that $\oint_C \frac{-y\,dx+x\,dy}{x^2+y^2}=2\pi$, Where C is any piecewise smooth Jordan curve enclosing the origin (0,0)

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+3-V-S-CBCS(MS)-Arts/Sc.(H)-Core-XII-Maths

2022

Time :As in Programme

Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer **ALL** *questions.*

GROUP - A

1. Answer all the questions. $1 \times 12 = 12$

(a) A vector space may have more than one zero vector. write True or false.

(b) A set consisting of a single non zero vector is ____________. (LI or LD)

(c) Write the dimension of the vector space $M_{m \times n}(F)$

(d) A is invertible if and only if L_A is invertible. write True or false.

(e) If V is isomorphic to W, then V^* is isomorphic to W^*. write True or false.

(f) Any polynomial of degree n with leading coeficient $(-1)^n$ is the Characterstic polynonial of some linear operator. write True or false.

(g) The adjoint of a linear operator is unique. True or false.

(h) Every self adjoint operator is normal. True or false.

(i) T is a linear map from V to W then $n(T)+r(T)=\lim(W)$. (T/F)

(j) $P_n(F)$ is isomorphic to $P_m(F)$ iff n = m write True or False

(k) Every vector space is isomorphic to its dual space, True or False

(l) Similar matrices always have the same eigenvalues write True or False.

GROUP - B

2. Answer any eight questions. $2 \times 8 = 16$

(a) In a vector space V, prove that $ox = 0$ for each $x \in V$.

(b) Write the Basis for the vector space of Complex numbers, over the field of real numbers.

(c) Define Basis for a vector space V.

(d) Is $T: R^2 \to R^2$ defined by $T(a_1, a_2) = (1, a_2)$ linear?

(e) Is the pair of vector space F^3 and $P_3(F)$ are ismorphic?

(f) Every diagonalizable linear operator on a non zero vector space has at least one eigenvalve. write True or False. Justify your Answer.

(g) Write cayley-Hamilton theorem for linear operators.

(h) Find the orthonormal set for the orthogonal set of non zero vectors $\{(1,1,0), (1,-1,1), (-1,1,2)\}$.

(i) Define orthonormal basis for an inner product space V.

(j) The gram - Schmidt orthogonalization process allows us to construct an orthonormal set from an arbitrary set of vectors. Justify your answer. by saying True or False.

GROUP-C

3. Answer any eight question. $3 \times 8 = 24$

(a) Prove that the span of any subset S of a vector space V is a subspace of V.

(b) Let V be a vector space having a finite basis. Prove that every basis for V contains the same number of vectors.

(c) Let V be a vector space with dimension n then prove that every linearly independent subset of V an be entended to a basis for V.

(d) Let V and W be vector spaces over a field F and let $T, U : V \to W$ be linear, prove that for all $a \in F$, $aT + U$ is linear.

(e) Let V and W be vector space and let $T : V \to W$ be linear and invertible then prove that $T^{-1} : W \to V$ is linear.

(f) Let V be a finite dimensional vector space with dual space V^*. Then proved that every ordered basis for V^* is the dual basis for some basis for V.

(g) Let $A \in M_{n \times n}(f)$. Then prove that a scular λ is an ligenvalue of A iff $\det(A = \lambda I_n) = 0$.

(h) Let T be a linear operator on R^3 defined by $T(a,b,c)=(-b+c,a+c,3c)$ find the T - cyclic subspace guranted by (1, 0, 0).

(i) Let $A \in M_{m\times n}(F)$. Then prove that rank $\left(A^{*}A\right)=$ rank (A)

(j) Prove that a linear operator T on G finite dimensional vector space V is diagonalizable of and only of V is the direct sum of the ligenspeces of T.

GROUP-D $7 \times 4 = 28$

4. Let V and W be vector space over F and suppose that $\{v_1, v_2, \ldots\ldots. v_n\}$ is a basis for V. For $w_1, w_2, \ldots\ldots. w_n$ in W, there exists exactly one linear transformation $T: V \to W$ such that $T(v_i)=w_i$ for $i=1,2,\ldots\ldots,n$ prove it.

Or

If a vector space V is generated by a finite Set S, then prove that some subset of S is a basis for V. hence V has a finite basis.

5. Let V and W be finite-dimensional vector spaces with ordered basis β and γ respectivelly. Let $T: V \to W$ be linear. Then prove that T is invertible if any only if $[T]_\beta^\gamma$ is invertible. Furthermore $\left[T^{-1}\right]_\gamma^\beta = \left([T]_\beta^\gamma\right)^{-1}$.

Or

Suppose that V is a finite-dimensional vector space with order basis $\beta=\{x_1, x_2, \ldots\ldots x_n\}$ let $f_i \,(1 \le i \le n)$ be the ith coordinate function with respect to β as just defined and let $\beta^* = \{f_1, f_2, \ldots f_n\}$. Then prove that β^* is an ordered basis for V^*, and for any $f \in v^*$ we have $f = \sum_{i=1}^{n} f(xi) f_i$.

6. Let $A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}$ Test A for diagonalizability and if A is diagonalizable, find an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ = D$.

Or

In R^4, let $w_1 = (1,0,1,0)$, $w_2 = (1,1,1,1)$ and $w_3 = (0,1,2,1)$, prove that $\{w_1, w_2, w_3\}$ is linearly independent, use Gram-Sehnidt process to compute the orthogonal vectors v_1, v_2, v_3 then normalize these vectors to obtain an orthonormal set.

7. Let V be a finite-dimensional inner product space, and let T be a linear operator on V. Then prove that there exist a unique function $T^* : V \to V$ such that $\langle T(x), y \rangle = \langle x, T^*(y) \rangle$ for all $x, y \in V$ furthermore T^* is linear.

Or

Let T be a linear operator on a finite-dimensional complex inner product space V. Then prove that T is normal if and only if there exists an orthonormal hasis for V consisting of ligenvectors of T.

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+3-V-S-CBCS(MS)-Arts/Sc.(H)-DSE-I-Maths

2022

Time :As in Programme

Full Marks : 80

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*Answer **ALL** questions.*

PART - I

1. Answer all the questions. $1 \times 12 = 12$

(a) Any solution to the LPP which also stisfies the non-negative restrictions of the problem is called ________.

(b) The non-negative variable which added to the Constraints of General LPP of type $\sum_{j=1}^{n} a_{ij}\, xj \leq bi$ called ________.

(c) The LPP of the form Max $z = cx, s.t\ Ax \leq b, x \geq 0$ is called ________ form.

(d) The Set of feasible solutions to an LPP forms ________ sel.

(e) If one or more of the basic variables vanish, a basic solution to the system $Ax = b$ is called ________.

(f) If the i[th] constraints of a primal (maxinisation) is equality, then the dual minimisation. variable y_i is ________

(g) The dwal of the LPP minimize $z = cx, s.t\ Ax \geq b, x \geq 0$ is ________.

(h) If the primal has infeasible solution then the dual has ________ solution.

(i) In a transportation problem with 4 supply points and 5 demand points, how many number of constraints are required in its formulation?

(j) The number of basic variables in an 5×4 transport table are __________.

(k) When the saddle point exists in a game?

(l) If the value of the game is zero, then the game is called __________.

PART - II

2. Answer any eight questions. $2 \times 8 = 16$

(a) Define Zero-Sum game?

(b) Write two assumptions made in game theory?

(c) Explain mixed strategy in short.

(d) Write mathematical formulation of an assignment problem.

(e) What is degeneracy in a transportation problem.

(f) Write a necessary and sufficient condition for the existance of a feasible solution to a transportation problem.

(g) State weak duality theorem.

(h) Write fundmental theorem of duality.

(i) Define basic feasible solution.

(j) Howmany basic feasible solutions are there to a given system of 3 simultaneous linear equations in 4 unknowns.

PART - III

3. Answer any eight question. $3 \times 8 = 24$

(a) Show that the following system of linear equations has degenerate solution

$2x_1 + x_2 - x_3 = 2, 3x_i + 2x_2 + x_3 = 3$

(b) Prove that any convex combination of k different optimum solutions to a LPP is again an optimum solution to the problem.

(c) Establish the difference between feasible solution, basic feasible solution and degenerate basic feasible solution.

(d) Write the dual of the LPP.

Min $z = 4x_1 + 6x_2 + 18x_3$ subject to

$x_1 + 3x_2 \geq 3, x_2 + 2x_3 \geq 5$ and $x_j \geq 0 (j = 1, 2, 3)$

(e) The dual of the dual is the primal. Prove it.

(f) Write down the symmetrical form of Primal-dual pair.

(g) State and prove the necessary and sufficient condition for the existance of a feasible solution to a transporation problem.

(h) Explain the difference between transportation problem and an assignment problem.

(i) Solve the game

		B I	B II	B III
A	I	6	8	6
	II	4	12	2

(j) For what value of λ the game with following pay-off matrix is strictly determinable

Player A		B_1	B_2
	A_1	2	6
	A_2	-2	λ

Player B

PART - IV $7 \times 4 = 28$

4. Use Big M method to

Minimize $z = 4x_1 + 3x_2$

S.t $2x_1 + x_2 \geq 10, -3x_1 + 2x_2 \leq 6$

$x_1 + x_2 \geq 6,\ x_1, x_2 \geq 0$

Or

Prove that if a LPP has a feasible solution then it also has a basic feasible solution.

5. Using duality solve the following

Maximize $z = 3x_1 + 2x_2$

subject to $x_1 + x_2 \geq 1,\ x_1 + x_2 \leq 7,\ x_1 + 2x_2 \leq 10,\ x_2 \leq 3$

$x_1, x_2 \geq 0$

Or

State and prove complementary slackness theorem.

6. Consider the following transportation problem

Origin	Destination D_1	D_2	D_3	D_4	Availability
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
Requirement	20	40	30	10	100

Determine an initial basic feasible solution using vogel's approximation method.

Or

Solve the following assignment problems.

$$\begin{array}{c} \\ I \\ II \\ III \\ IV \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 4 & 6 & 3 \\ 9 & 7 & 10 & 9 \\ 4 & 5 & 11 & 7 \\ 8 & 7 & 8 & 5 \end{bmatrix} \end{array}$$

7. Solve the following 2×3 game graphically

Player B

$$\text{Player A} \quad \begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}$$

Or

Solve the game whose pay-off matrix is given by

Player B

$$\text{Player A} \quad \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{bmatrix} \end{array}$$

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+3-V-S-CBCS(MS)-Arts/Sc.(H)-DSE-II-Maths

2022

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Full Marks : 80

The figures in the right-hand margin indicate marks.

Answer **ALL** *questions.*

PART - I

1. Answer all the questions. $1 \times 12 = 12$

(a) In how many ways can six tosses of coin yield two heads and four tails?

(b) Define continuous random variable.

(c) Write the relationship probability density function f(x) and probability distribution function F(x)

(d) What is var (b) if b is a constant?

(e) Define moment generating function of a discrete random variable x

(f) Find the range of the list of values 13, 18, 13, 14, 13, 16, 14, 21, 13

(g) The binomial distribution b(x,n,v) reduces bernouli's distribution when n=__________.

(h) If $f(x)=2x,\ 0<x<1$ and zero elsewhere is the pdf of random variable *x*, then find $E\left(\frac{1}{x}\right)$

(i) Define joint probability distribution of 'n' random variables

(j) Define sample mean of random variables.

(k) Write the statement of central limit theorem.

(l) Define population distribution.

PART - II

2. Answer any eight questions of the following. $2 \times 8 = 16$

(a) If A and A' are complementary events in a sample space then prove that $P(A') = 1 - P(A)$.

(b) Cheek whether the function given by $f(x) = \frac{x+2}{25}$ for $x = 1,2,3,4,5$ can serve as the probability distribution of a discrete random variable.

(c) If x is a discrete random variable then prove that $F(a) \leq F(b)$ for $a < b$ where $F(x)$ is the distribution function

(d) Prove that $E(ax+b) = aE(x)+b$ where a and b are constants.

(e) Write the first three moments about the mean.

(f) Define product moments about the origin of the random variables X and Y for both discrete and continuous case.

(g) What is the formula of mean and variance of discrete uniform distribution?

(h) Define Bivariate normal distribution

(i) Write the probability distribution of sum of 'n' independent random variables $x_1, x_2 ..., x_n$ having poisson distributions $\lambda_1, \lambda_2, ..., \lambda_n$.

(j) Define F distribution

PART - III

3. Answer any eight question. $3 \times 8 = 24$

(a) Find a formula for the probability distribution of the total number of heads obtained in four tosses of a balanced coin.

(b) A family has two children. What is the probability that both the children are boys given that at least one of them is a boy?

(c) If A and B are independent, then prove that A and B' are independent.

(d) If a random variable x has the variance σ^2 then prove that var $(ax+b)=a^2\sigma^2$.

(e) Find the mean of the random variable X be a number when a die is thrown once

(f) let x be a number selected at random from a set of numbers $\{51, 52, ..., 100\}$. Find $E\left(\frac{1}{x}\right)$

(g) Find the mean of Gamma distribution.

(h) Prove that if X has a normal distribution with the mean μ and standard deviation σ then $z=\frac{x-\mu}{\sigma}$ has the standard normal distribution.

(i) In 16 one-hour test runs, the gasoline consumption of an engine averaged 16.4 gallons with a standard deviation of 2.1 gallons. Test the claim that the average gasoline consumption of this engine is 12.0 gasolons per hour.

(j) If $x_1, x_2 ....., x_n$ are independent random variables having standard normal distributions, then prove that $Y=\sum_{i=1}^{n} X_i^2$ has the chi-square distribution with $v = n$ degrees of freedom.

PART - IV $7 \times 4 = 28$

4. State and prove Baye's theoren.

Or

Given the joint probability density

$$f(x,y) = \begin{cases} 4xy, & 0<x<1,\ 0<y<1 \\ 0 & \text{elsewhere} \end{cases}$$

find the marginal densities of x and y and the conditional density of X given Y = y

5. Given that X has the probability distribution $f(x) = \frac{1}{8} C(3,x)$ for $x = 0, 1, 2$ and 3, find the moment generating function of this random variable and use it to determine μ_1^1 and μ_2^1.

Or

A lot of 12 television sets include 2 with white cords. If 3 of the sets are choosen at random for shipment to a hotel, how many sets with white cords can the shipper expect to send the hotel?

6. Show that mean and variance of poisson distribution are equal.

Or

Find the mean and variance of Gamma distribution.

7. If the probability density of X is given by

$$f(x) = \begin{cases} 6x(1-x), & 0<x<1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability density of $y = x^3$

Or

If the joint density of x_1, x_2 and x_3 is given by

$$f(x_1,x_2,x_3) = \begin{cases} (x_1+x_2)e^{-x_3}, & 0<x_1<1,\ 0<x_2<1, x_3>0 \\ 0 & \text{elsewhere} \end{cases}$$

Find the regression equation of x_2 on x_1 and x_3

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