NUMERICAL ANALYSIS

2022

Time :As in Programme

Full Marks : 80

The figures in the right-hand margin indicate marks.

*Answer **all** questions.*

PART - I

1. Answer all the questions : 1 x 8

a) Is the interval [0.5,1] contains a root of the equation $f(x) = \cos x - xe^{x} = 0$ write yes or no.

b) Errors may occur in performing numerical computation on the computer is called ______.

c) The Gausss Jordan method reduces a original matrix into a ________.

d) A quadratic equation $x^4 - x - 8 = 0$ is defined with an initial guess of 1 and 2. The approximated value of x_2 using secant method is _____.

e) The order of convergence of Newton Raphson method is ______.

f) Gauss - Seidel methods gives result faster when the pivotal element are _______.

g) Lagrange's interpolation formula can be used only for equally spaced nodes. write true or false.

h) In Simpson's ($^1/_3$)rd Rule the number of intervals is _____. (even /odd).

(Turn Over)

PART - II

2. Answer any eight questions 1.5 x 8

   a) What do you mean by round off error?

   b) Write the intermediate value theorem to find the initial approximation roots?

   c) Define the rate of convergence of iterative method to find the roots of an equation.

   d) What is partial pivoting.

   e) Define interpolating polynomial.

   f) Write the formula for average operator.

   g) Write the formula for error in Simpson's $^1/_3$ role.

   h) Find the approximate value of $\int_0^1 \frac{dx}{1+x}$ using trapezoidal rule.

   i) Write the formula for $\ell_i(x_i)$ if the number of nodes are i=0, 1, 2, 3, 4, 5,n-1 in lagrange interpolation.

   j) What is extrapolation?

PART - III

3. Answer any eight questions. 2 x 8

   a) Find the smaller root of the equation $x^2 - 400x + 1 = 0$ using four digit arithmetic.

   b) Obtain 3 iterations of the Prisection method to obtain a root of the equation $f(x) = \cos x - xe^x = 0$.

   c) Derive the formula for Secant method to find the approximation to the root of an equation.

   d) Find the step size 'h' that can be used in the tabulation of $f(x) = \text{Sin } x$ in [1,3]. So that the linear interpolation will be correct to four decimal places.

e) Prove that $M = \left(1+\frac{\delta^2}{4}\right)^{1/2}$

f) Solve the equation $10x_1 - x_2 + 2x_3 = 0$

g) Find the unique polynomial of degree 2 or less such that

$f(o) = 1, f(1) = 3, f(3) = 55.$

h) Solve $10x_1 - x_2 + 2x_3 = 4, x_1 + 10x_2 - x_3 = 3$

$2x_1 + 3x_2 + 20x_3 = 7$ by using Gauss elimination.

i) Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$ using simpson's rule and obtain a bound for the error.

j) Using the data Sin (0.1)=0.09983 and Sin (0.2)=0.19867, Find an approximate value of Sin (0.15) by lagrange's interpolation.

PART - IV

7 x 4

4. Use the Secant and Regula-falsi methods to determine the root of the equation $\text{Cos}\, x - xe^x = 0$

OR

Obtain a Second degree polynomial approximation to $f(x) = (1+x)^{1/2}, x \in [0,1]$ Using the Taylor's Series expansion about x = 0. Use the expansion to approximate f (0.05) and find a bound of the truncation error.

5. Prove that Newton-raphsons method is Quadratically Convergent.

OR

Find the inverse of the matrix

$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ *Using the* Gauss - Jordan Method.

6. Using the Newton's backward difference interpolati
Construct the interpolating polynomial that fits the data

x	0.1	0.3	0.5	0.7	0.9	1.1
f(x)	-1.699	-1.073	-0.375	0.443	1.429	2.631

Estimate the value of f(x) at x = 0.6 and x = 1.0

OR

Given the following values of f(x) and f'(x)

x	f(x)	f'(x)
-1	1	-5
0	1	1
1	3	7

estimate the values of f (-0.5) and f (0.5) using Hermite interpolation.

7. Find the remainder of the Simpson's three eight rule

$$\int_{x_0}^{x_1} f(x)dx = \frac{3h}{8}[f(x_0)+3f(x_1)+3f(x_2)+f(x_3))$$

for equally spaced points $x_i = x_0 + ih; i = 1,2,3$

Evaluate $\int_0^1 \frac{dx}{1+x^2}$ Using Simpson's three eight rule. Compare the exact solution.

OR

Evaluate $\int_0^1 \frac{dx}{1+x}$ using

i) Composite trapezoidal rule

ii) Composite Simpson's rule with 2, 4, 8 equal subintervals.

2022

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*Answer **all** questions.*

Part - I

1. Answer all the questions : [1 x 12=12

a) Let $x=(x_1, x_2, \ldots, x_n)$ $y=(y_1, y_2, \ldots, y_n)$ be any two points of R^n then the inequality

$$\sum_{k=1}^{n}|x_k y_k| \le \left(\sum_{k=1}^{n}|x_k|^2\right)^{1/2}\left(\sum_{k=1}^{n}|y_k|^2\right)^{1/2}$$ is called ___.

b) The distance between

$$A=\{(x,y)\in R^2; y=\frac{1}{x}, x\neq 0\}$$

$$and\ B=\{(x,y)\in R^2, y=0\}$$

is _____.

c) If $A=\phi$ and $B=R$ then $A^0\cup B^0=$ _____

d) Let $A=\{\frac{1}{n}, n\in N\}$ is a subset of Euclidean metric space (R, d) then derived set of A is _____.

e) Every subset of a discrete metric space is open as well as closed. Write true or false.

f) If f and g are continuous real valued functions on the metric space X. Then the Set $A = \{x \in x : f(x) < g(x)\}$ is closed. write true or false.

g) Let $f : X \to Y$ be a continuous map. If $\{x_n\}$ be a cauchy sequence in (X, d) then $\{f(x_n)\}$ is a cauchy sequence in (Y,P) write true or false.

h) If $f : R_U \to R_U$ defined by $f(x) = x^2$ uniformly continuous. Write yes or no.

i) Let (x, d_x) & (Y, d_y) be two metric space and $f : X \to Y$ is open. Is 'f' continuous?

j) The mapping $f(x) = 2x$ where $f : [0,1] \to [0,2]$ with usual absolute value metric is a homomorphism. Write true or false.

k) Let $T : X \to X$ be a mapping, defined by $T_x = x$, *then* $x \in X$ is called ______.

l) In a metric space, any two disjoint sets are always separated. Write true or false.

Part - II

2. Answer any eight questions 2 x 8=16

a) Define Euclidean metric on R^n.

b) Find $S_r(x)$ in a discrete metric space (X, d).

c) Give an example to show that intersection of an infinite number of open sets need not be open.

d) Let (X,d) be a metric space and $x \in X$, then define local base at x.

e) Is the subsets $\{3n+1, n \in Z\}, \{3n, n \in Z\}$ and $\{3n+2, n \in Z\}$ forms an open cover of Z. Justify your answer.

f) Every subset must be either of category I or of Category II. Write true or false.

g) When a function $f: X \rightarrow Y$ is said to be an isometry.

h) Define equivalent matrics on a metric space X.

i) Is T has fined point where $T: X \rightarrow X$ and X is a complete metric space.

j) Define compact metric space.

Part - III

3. Answer any eight questions. [3 x 8=24

a) Prove that a Convergent Sequence in a metric space is a Cauchy Sequence.

b) Prove that in any metric space (X,d) each open ball is an open set.

c) Let (X,d) be a metric space and A, B be subsets of X. Then prove that $(A \cup B)^0 \supseteq A^0 \cup B^0$.

d) Prove that in any metric space, there is a countable base at each point.

e) If $X = Y_1 \bigcup Y_2$ *and* Y_1 is of category I while X is of category II, then prove that Y_2 must be category II.

f) Let $(X, d_x), (Y, d_y)$ *and* (Z, d_z) be metric spaces and let $f: X \rightarrow Y$ *and* $g: Y \rightarrow Z$ be continuous. Then prove that g of is a continuous map of X into Z.

g) Let $f(x) = \text{Sin } (\frac{1}{x}), x \in R - \{0\}$ Prove that the function f cannot be extended to a continuous function on R.

h) Let A be a subset of the metric space (X,d). Define $f(x) = d(x, A) = \inf\{d(x, y) : y \in A\}, x \in X$ Prove that f is uniformly continuous over X.

i) Prove that two metrics d_1 and d_2 on a non-empty set X are equivalent if there exist a constant k such that

$$\frac{1}{k}d_2(x, y) \le d_1(x, y) \le k\ d_2(x, y), \forall x, y \in X.$$

j) Let (x, d) be a metric space, prove that if (x,d) is disconnected then $\exists$ two empty disjoint subsets A and B, both open in X such that $X = A \cup B$.

Part - IV

4. Prove that the space ℓ^p is complete [7 x 4=28

OR

Let (x,d) be a metric space and F be a subset of X. Prove that F is closed in X if and only if F^c is open in X.

5. Prove that any complete metric space is of catagory II.

OR

Let (x, d) be a metric space and $Y \underline{C} X$ prove that if X is separable then Y with the induced metric is separable too.

6. Prove that $f: X \to Y$ is continuous on X if and only if $f^{-1}(G)$ is open in X for all open subsets G of Y.

OR

Let (X, d_x) *and* (Y, d_y) be two metric spaces and $f: X \to Y$ be uniformly continuous. If $\{xn\}_{n\ge1}$ is a Cauchy Sequence in X, then prove that $\{f(xn)\}_{n\ge1}$ is a Cauchy sequence in Y.

7. Let $T: X \to X$ be a contraction of the complete metric space (X,d). Then prove that T has a unique fixed point.

OR

Let (X,d) be a metric space. Then prove that the following statements are equivallent.

i) (X,d) is disconnected

ii) There exist a continuous mapping of (X,d) onto the discrete two element space (X_o, d_o).

+3-IV-S-CBCS(MS)-Arts/Sc(H)-Core-X-Math

2022

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Full Marks : 80

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*Answer **all** questions.*

Part - I

1. Answer all questions [1 x 12 = 12

a) Define Boolean ring?

b) Give an example of zero divisor ring

c) Z is not ideal of Q (True / False).

d) Find no of homomorphism from $Z_6 \times Z_{12}$

e) $\forall\ n\ Z / nZ$ is an integral domain (True / False)

f) Every maximal Ideal is not prime (True / False)

g) Is $x^2 + x + 4$ is irreducible over Z_{11} (Yes / No)

h) Find all zeros of $\frac{1}{2}x^2 - \frac{5}{2}x + 3 \in Q\ [x]$

i) Every PID is a ED (True/False)

j) If P(x) is irredncible polynomial over a field $f[x] / < P(x) >$ is a field (True / False).

k) For any integer 'n' the set nz={0, ±n, ±2n...} is an ideal of Z (True / False)

l) Define Kernel of ring.

Part - II

2. Answer any eight questions [2 x 8 =16

a) Show that ring Z_p is commutative ring with unity.

b) Show that if 'n' is an integer and 'a' is an element from a ring then prove that $n(-a) = -(na)$.

c) Is $\phi : z \to 2z$ such that $\phi(x) = 2x$ is a homomorphism.

d) Find unity and zero divisor of Z_{15}.

e) Prove that Z_n is not a field when 'n' is not prime.

f) Find char (Z6)?

g) Show that $x^4 + 4$ has four zeros in Z_5.

h) Let $\phi : R_1 \to R_2$ be homomorphism then show that $\phi(-a) = -\phi(a)$.

i) Show that commutative ring with the cancellation property has no zero divisor.

j) Find the principal ideals of Z_{10} generated by $\bar{3}$ *and* $\bar{5}$

Part - III

3. Answer any eight questions. [8 x 3 = 24

a) Prove that a ring can't have at most one unity.

b) Find all solutions of $x^2 - x + 2 = 0$ over $Z_3[i]$.

c) Find all maximal Ideal of Z_n?

d) Suppose that R is a ring and $a^2 = a \forall a$. show that R is commutative.

e) Prove that every field is an integral domain.

f) Show that the ring $z[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in z\}$ is an integral domain.

g) If $\phi : Z_6 \to Z_3 : \phi(n(\bmod 6)) = n(\bmod 3)$ show that ϕ is a ring homomerphism.

h) The ring Z_p is commutative ring with unity.

i) In an integral domain every prime is irreducible prove it.

j) Let 'F' be a field then F[x] is a UFD.

k) Ring $(Z_p = 0,1,2....p-1,+P,\times P)$ is an integral domain if P is prime prove it?

Part - IV

Answer all questions. [7 x 4 = 28

4. Prove that a finite integral domain is a field.

OR

Let R be a commutative ring with unity and let A be an ideal of R. Then R/A is a field iff A is maximal.

5. State and prove first isomerphism theorem for rings.

OR

Let R be a ring with unity e. The mapping $\phi : Z \to R$ given by $n \to ne$ is a ring homomorphism.

6. Let 'F' be a field and let $P(x) \in F(x)$. Then <P(x)> is a maximal ideal in F[x] iff p(x) is irreducible over F.

OR

The product of two primitive polynomials is primitive.

7. Every principal ideal domain is a unique factorization domain.

OR

The ring of Gaussian integers

Z [i]={a+ib | a, b ∈ Z}is an Euclidean domain with $d(a+ib) = a^2 + b^2$ prove it?

2022

Time :As in Programme

Full Marks : 60

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*Answer **all** questions.*

PART-I

1. Answer all the questions. 1x8

   a. The enthalpy change for an endothermic reaction is always ____.

   b. For the given reaction $C(s)+CO_2(g)\leftrightarrow 2CO(g)$, Kc=?

   c. The substance which enhances the efficiency of a catalyst is called ____.

   d. Write down the expression for pKa = ?

   e. Aldehydes having ____ hydrogen atom undergo Aldol condensation.

   f. Write down the formula of Tollen's reagent.

   g. In chloro benzene -Cl is ____ directing in nature.

   h. Heat of formation of natural occuring elements is ____.

PART-II

2. Answer any eight within two to three sentences 1.5x8

   a. What is an isolated system ?

(Turn Over)

b. What is the relation between enthalpy change and entropy change ?

c. What is the effect of change of pressure for the following reaction

$$H_2(g) + I_2(g) \leftrightarrow 2HI(g)$$

d. Define solubility product.

e. Explain why AgCl is less soluble in NaCl solution than water.

f. What is the order of reactivity of the following substances towards SN^1 reaction.

CH_3Br, CH_3CH_2Br, $(CH_3)_2CHBr$, $(CH_3)_3Br$

g. Explain why haloarenes are less reactive than haloalkenes.

h. Explain why t-butyl alcohol is more soluble than n-butyl alcohol in water.

i. What are the reagents used in nitration of benzene ?

j. Explain the effect of temperature on the pH of a solution.

PART-III

3. Answer any eight of the following (in maximum 75 words.)

2x8

a. Define integral enthalpy of solution.

b. What is free energy of a system ? How it affects the spontaneity of a reaction ?

c. What are the factors affecting bond energy of a compound ?

d. What is the pH of 10^{-5} M NaOH solution ?

e. Define buffer solution. Explain with example.

f. Calculate the heat of reaction of $CH_2Cl_2(g) \rightarrow C(g) + 2H(g) + 2Cl(g)$

Given that bond energy of C-H=414 KJ, C-Cl=328 KJ

g. What happens when formaldehyde reacts with CH_3MgBr in acid medium ?

h. What happens when phenol reacts with $CHCl_3$ and NaOH and 60°C ?

i. Explain why phenol is more soluble in water than toluene.

j. What do you mean by hydrolysis of a salt ? Explain.

PART-IV

Answer within 500 words each. 6x4

4. a. State and explain Hess's law of constant heat summation.

b. Calculate the standard heat of formation of n-butane, given that standard enthalpy of combustition of n-butane, C(graphite) and $H_2(g)$ are -2878.5, -393.5 and -285 $KJmol^{-1}$.

OR

a. Derive Kirchoff's equation.

b. Consider the reaction $N_2 + 3H_2 \leftrightarrow 2NH_3$, $\Delta H^0 = -22.08$ Kcal at 25°C. What is the ΔH^0 at 35°C ? Cp of N_2, H_2 and NH_3 are 6.8, 6.77 and 8.86 cal K^{-1} mol^{-1}.

5. State and explain Lechatelier's principle. Discuss the effect of change in concentration, temperature and pressure at equilibrium with examples.

OR

What is the law of chemical equilibrium ? Derive the relation between Kc, Kp and Kx thermodynamically.

6. a. State and explain Ostwald's dilution law.

b. A 0.001 M solution of acetic acid dissociates to an extent of 1.5% at 300K. Determine the dissociation constant at 300K.

OR

What is buffer capacity ? Derive the expression for pH of acidic buffer solution.

7. Write notes on :

a. Oppeneaur oxidation

b. Reimer-Tiemann reaction

c. Benzoin condensation

OR

Write notes on :

a. Aldol condensation

b. Wolff Kishner reduction

c. Pinacol-Pinacolonne rearrangement

2022

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*Answer **all** questions.*

PART-I

1. Answer all the questions. 1x12

a. When files are arranged area-wise then it is ____ classification.

b. Record retention is needed as a :

i. piece of evidence

ii. for making payments.

iii. part of business activity.

c. Office staff handles various activities in :

i. marketing

ii. production

iii. office

d. Office work is ____ (productive / unproductive)

e. Communication from subordinate to a superior is called ____ communication.

f. A ____ is the subject matter of communication.

(Turn Over)

g. Look of attention is a part of _____ barrier in communication.

h. Creative skills of a supervisor help in getting new _____.

i. Political skills help to acquire _____ to achieve objectives.

j. A report is a _____ of information.

k. Incoming mail means :

i. correspondence with office

ii. letter sent out

iii. letter coming from outside

l. In which type of filling system letters are kept in standing position.

PART-II

2. Answer any eight within two to three sentences 2x8

a. What is centralised system of filing ?

b. Define record management.

c. Name subsidiary functions of office.

d. Why is proper span of supervision necessary ?

e. What do you mean by 'structure of a report' ?

f. State any three features of a joint stock company.

g. Discuss the concept of a joint venture.

h. What do you mean by public enterprises ?

i. Name subsidiary functions of office.

j. What is meant by office work ?

PART-III

3. Answer any eight of the following (in maximum 75 words.)

3x8

a. Explain pigeon hole filling system.

b. Write the features of leadership.

c. Explain the state of emotional intelligence.

d. What is gestural communication ?

e. State any three principles of effective office communication.

f. Give briefly the responsibilities of a supervisor.

g. Can a report be prepared without purposes - explain.

h. What do you mean by indexing ?

i. What do you mean by motivation ?

j. What are the need of having index system ?

PART-IV

Answer within 500 words each. 7x4

4. What do you understand by size of office ? Which factors determine the size of office ?

OR

What is meant by office staff ? What types of skills should be acquired by the office staff ?

5. Define Indexing. Give its objectives and importance.

OR

Define mail. Describe the advantages of decentralised mail system.

6. Define a report. Discuss the 5w-h plan for writing a report.

OR

How can barriers in communication be overcome ?

7. Define leadership. Describe the nature of autocratic and democratic styles of leadership.

OR

What do you mean by motivation ? Write or explain the process of motivation.